MODELING NONLOCAL MASS TRANSFER OF A DISPERSED IMPURITY IN TURBULENT FLOWS OF A GAS SUSPENSION

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The effect of nonlocal phenomena occurring with the motion of inertial particles on the rate of mass transfer of the dispersed phase is studied on the basis of a closed expression constructed for the probability density function determining the transport of these particles in a turbulent nonuniform flow.

The main types of models are currently used to determine the intensity of pulsative motion in turbulent nonuniform flows. The first type is based on the hypothesis of local equilibrium, when the intensity of turbulent transport at a given point in space is determined by the fluctuational and averaged characteristics of the turbulent flow at this point. The local-equilibrium approximation is valid when the characteristic three-dimensional scale of turbulence is smaller than the scale of measurement of the turbulent field's averaged parameters. Models of the second type, based on nonlocal transport, are used to describe pulsative transfer in essentially nonuniform turbulent flows. Here, the turbulence scale is comparable to the characteristic scales of the flow itself. In this case, the rate of turbulent transport at a given point in space depends integrally on the flow characteristics in the neighborhood of this point [1-3].

The spatial scale of turbulence is proportional to the time scale of the turbulent pulsations. In the case of the motion of a dust-laden turbulent flow, the time scale of the turbulent pulsations of the disperse phase of fine particles – the dynamic relaxation time of which is shorter than the turbulence time scale – coincides with the characteristic time of turbulent pulsation of the carrier flow [4]. The rate of pulsative motion of the disperse phase at a certain point of the flow depends on the pulsative and averaged characteristics of the carrier phase at the given point. Here, the validity of the local and nonlocal model for calculation of the rate of turbulent transport of the carrier phase is determined by the ratio of the scales of the flow itself. For intertial particles, whose dynamic relaxation time exceeds the time scale of the turbulent pulsations of the fluid phase, the lifetime of the turbulent pulsations of the disperse phase is determined by the dynamic relaxation of the particles. The spatial scale of the pulsations of the disperse phase $L_p \simeq \tau\sigma^{1/2}$ may significantly exceed the characteristic scales of the carrier flow. The rate of pulsative motion of the disperse phase at a given point of the flow depends on the intensity of the turbulent pulsations of the particles in a neighborhood with dimensions on the order of L_p and its center at the chosen point. In this case, regardless of the method used to describe fluctuational transport of the carrier phase (local or nonlocal), the turbulent transport of the dispersed impurity can be studied only within the framework of nonlocal models.

The manifestation of nonlocal effects in nonuniform turbulent flows is important to consider. In contrast to the fluctuations of the velocity of the fluid phase, the intensity of the pulsative motion of a flow of inertial particles in the fluid does not vanish on the walls of the channel in which the flow takes place. Intensive turbulent motion of the particles in the wall region results in effective mass transfer of the impurity (deposition of particles) and transfer of momentum (averaged velocity slip of the phases) to the walls. Semiempirical descriptions of inertial transfer of the pulsative energy of the particles to the walls are based on different modifications of the model of the inertial path of particles [5-7]. Within the framework of this model, the intensity of the pulsative motion of an impurity on the wall of a channel is equated to the turbulence energy of the particles in the flow at the distance L_p from the wall.

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Along with nonlocal effects which arise in the motion of inertial particles in turbulent flows, calculation of the fluctuational characteristics of the disperse phase requires consideration of the effect of intersection of the trajectories of the particles and fluid moles due to the averaged velocity slip of the phases.

Using the approximation of isotropic turbulence, investigators studied nonlocal transport and the particle-trajectory intersection effect by both theoretical [4, 8-13] and experimental [14-16] means. It should be noted that the nonlocal effects in this case were due to the fact that the scale of particle pulsations L_p was greater than the characteristic size of the small-scale eddies which form an energy-containing turbulent mole. In [8, 9], turbulent pulsations of the velocity of the carrier phase were modeled by a Gaussian random field and it was assumed that the Lagrangians of the pulsations of particle velocity constituted a normal random process. The authors of [8, 9] found self-consistent expressions for the square of the fluctuations of particle velocity in a Lagrangian representation and the particles' eddy diffusion coefficient. These expressions consider both the different degrees of involvement of particles in pulsative motion associated with fine-scale turbulence and the reduction in turbulent diffusion of the particles due to intersection of the trajectories. Proceeding within the framework of a renormalized perturbation theory, the author of [10] studied the intensity of pulsative particle motion in an Eulerian approximation. With the use of different approximations of the autocorrelation function for pulsations of the velocity of the carrier phase along the particle trajectories, the studies [4, 11-13] examined the effect of particle inertia and averaged phase slip on the fluctuational characteristics of the turbulent motion of a dispersed impurity in an approximation in which the carrier flow was assumed to be locally isotropic. Calculations of the turbulent dispersion of particles in a mixing field were performed in [14] on the basis of a system of equations for the second moments of the velocity pulsations of noninertial particles with averaged phase slip.

Here, with the assumption that the field of turbulent fluctuations of the carrier phase is Gaussian in character, we find closed expressions for the density function for the transport of one or two inertial particles in space. We study the effect of particle inertia and average phase slip on the intensity of turbulent pulsations and eddy diffusion coefficient of particles in the case of uniform and nonuniform turbulence.

1. The equation of motion of a single particle located in a turbulent flow in a body-force field and undergoing Brownian motion has the form

$$\frac{dV_{pi}}{dt} = \frac{1}{\tau}(\langle U_i(\mathbf{R}_p(t),\ t)\rangle + u_i(\mathbf{R}_p(t),\ t) + W_i + f_i(\mathbf{R}_p(t),\ t) - V_{pi}), \tag{1}$$

$$\frac{dR_{pi}}{dt} = V_{pi},$$

where the autocorrelaton function for the Brownian pulsations of particle velocity is written as follows:

$$\langle f_i(\mathbf{x}_1,\ t_1) f_j(\mathbf{x}_2,\ t_2)\rangle = \delta_{ij}\delta(\mathbf{x}_1 - \mathbf{x}_2)\,\delta(t_1 - t_2)\,D_0. \tag{2}$$

Assuming the existence of Gaussian turbulence fields representing the fluctuations of carrier-phase velocity and the velocity of the Brownian displacements, we obtain an expression for the particles' eddy diffusion coefficient [17]

$$D_{ij}^p = \tau\sigma_{ij} + \int d\mathbf{x}_1 \int_0^t dt_1 \left[1 - \exp\left(-\frac{t - t_1}{\tau}\right)\right] \times \times \langle u_i(\mathbf{x},\ t) u_j(\mathbf{x}_1,\ t_1)\,\delta(\mathbf{x}_1 - \mathbf{R}_p(t_1))\rangle. \tag{3}$$

We assume that the turbulent motion of the particles is due to fluctuations of viscous drag and Brownian oscillations. We use the equation of motion of a single particle (1) to find an expression for calculating the rate of pulsative motion of the disperse phase

$$\sigma_{ij}\langle N(\mathbf{x},\ t)\rangle = \langle \delta(\mathbf{x} - \mathbf{R}_p(t))\, v_{pi}(t)\, v_{pj}(t)\rangle = = \frac{D_0}{\tau}\langle N\rangle + \frac{1}{\tau^2}\int d\mathbf{x}_1 \int d\mathbf{x}_2 \int_0^t dt_1 \exp\left(-\frac{t - t_1}{\tau}\right) \times \tag{4}$$

$$\times \int_0^t dt_2 \exp\left(-\frac{t-t_2}{\tau}\right) \langle u_i(\mathbf{x}_1, t_1) u_j(\mathbf{x}_2, t_2) \times \delta(\mathbf{x}_1 - \mathbf{R}_p(t_1)) \delta(\mathbf{x}_2 - \mathbf{R}_p(t_2)) \delta(\mathbf{x} - \mathbf{R}_p(t)) \rangle,$$

where $\langle N(\mathbf{x}, t)\rangle = \langle \delta(\mathbf{x} - \mathbf{R}_p(t))\rangle$ is the density function associated with finding a particle at the point $\mathbf{x}$ at the moment of time t.

It follows from Eqs. (3) and (4) that, in order to calculate the rate of pulsative motion (fluctuation velocity) and eddy diffusion coefficient of the particles, it is necessary to sum the carrier-phase fluctuation velocities calculated for those random trajectories of discrete particles that lead from points $\mathbf{x}_1$ and $\mathbf{x}_2$ at the moments of time t_1 and t_2 to point $\mathbf{x}$ at the moment of time t. To perform this summation, we introduce a density function which gives the probability of the transfer of a single particle from point $\mathbf{x}_1$, t_1 to point $\mathbf{x}$, t: $G_0(\mathbf{x}, t| \mathbf{x}_1, t_1)$. The integrands in (3) and (4) take the form

$$\begin{gathered} \langle u_i(\mathbf{x}, t) u_j(\mathbf{x}_1, t_1) \delta(\mathbf{x}_1 - \mathbf{R}_p(t_1)) \rangle = \langle u_i(\mathbf{x}, t) u_j(\mathbf{x}_1, t_1) G_0(\mathbf{x}, t|\mathbf{x}_1, t_1) \rangle, \\ \langle u_i(\mathbf{x}_1, t_1) u_j(\mathbf{x}_2, t_2) \delta(\mathbf{x}_1 - \mathbf{R}_p(t_1)) \delta(\mathbf{x}_2 - \mathbf{R}_p(t_2)) \delta(\mathbf{x} - \mathbf{R}_p(t)) \rangle = \\ = \langle N \rangle \langle u_i(\mathbf{x}_1, t_1) u_j(\mathbf{x}_2, t_2) G_0(\mathbf{x}, t|\mathbf{x}_1, t_1) G_0(\mathbf{x}, t|\mathbf{x}_2, t_2) \rangle. \end{gathered} \tag{5}$$

To calculate the function $G_0(\mathbf{x}, t| \mathbf{x}_1, t_1)$, we will examine the more general function $G_0{}^1(\mathbf{x}, \mathbf{V}, t| \mathbf{x}_1, \mathbf{V}_1, t_1)$. The latter is the density function for the transfer of particles in the phase space from the point $\mathbf{x}_1$, $\mathbf{V}_1$ to the point $\mathbf{x}$, $\mathbf{V}$ during the time $t - t_1$. Following an approach similar to [10] and using the equation of motion of a single particle (1), we obtain an expression for $G_0{}^1$ without allowance for Brownian fluctuations of particle velocity:

$$\begin{gathered} G_0'(\mathbf{x}, \mathbf{v}, t|\mathbf{x}_1, \mathbf{v}_1, t_1) = \delta\Big(\mathbf{x} - \mathbf{x}_1 + \tau(\mathbf{v} - \mathbf{v}_1) - \\ - (t-t_1)\mathbf{W} - (t-t_1)\langle \mathbf{U} \rangle - \int_{t_1}^t ds\mathbf{u}(\mathbf{R}_p(s), s)\Big) \times \\ \times \delta\left(\mathbf{v} - \mathbf{v}_1 \exp\left(-\frac{t-t_1}{\tau}\right) - \frac{1}{\tau}\int_{t_1}^t ds \exp\left(-\frac{t-s}{\tau}\right) \times \right. \\ \left. \times\, u(\mathbf{R}_p(s), s)\right), \quad \mathbf{v} = \mathbf{V} - \langle \mathbf{V} \rangle. \end{gathered} \tag{6}$$

It is evident from (6) that the displacement of a particle in space is composed of the inertial transport (a term proportional to the dynamic relaxation time of the particles), displacement of the particle with an averaged relative velocity due to the body force, and displacement of the particle together with the fluid. The velocity of a fluid particle is calculated on the basis of the trajectory of a solid particle [the last two terms in the argument of the first δ-function in (6)]. We will use $R_f{}^p(t - t_1)$ to represent the displacement of the fluid particle. Then we can write Eqs. (5) in the form

$$\begin{gathered} \langle u_i(\mathbf{x}, t) u_j(\mathbf{x}_1, t_1) G_0(\mathbf{x}, t|\mathbf{x}_1, t_1) \rangle = \\ = \langle u_i(\mathbf{x}, t) u_j(\mathbf{x}_1' - \mathbf{R}_f^p(t-t_1), t_1) G_1(\mathbf{x}, t|\mathbf{x}_1', t_1) \rangle, \\ \langle u_i(\mathbf{x}_1, t_1) u_j(\mathbf{x}_2, t_2) G_0(\mathbf{x}, t|\mathbf{x}_1, t_1) G_0(\mathbf{x}, t|\mathbf{x}_2, t_2) \rangle = \\ = \langle u_i(\mathbf{x}_1' - \mathbf{R}_f^p(t-t_1), t_1) u_j(\mathbf{x}_2' - \mathbf{R}_f^p(t-t_2), t_2) \times \\ \times G_1(\mathbf{x}, t|\mathbf{x}_1', t_1) G_1(\mathbf{x}, t|\mathbf{x}_2', t_2) \rangle, \end{gathered} \tag{7}$$

$$\begin{gathered} \mathbf{R}_f^p(t-t_k) = (t-t_k)\langle \mathbf{U} \rangle + \int_{t_k}^t ds\mathbf{u}(\mathbf{R}_p(s), s), \\ \mathbf{x}_k' = \mathbf{x}_k + \mathbf{R}_f^p(t-t_k), \quad k = 1, 2. \end{gathered} \tag{8}$$

Here, the function $G_1(\mathbf{x}, t| \mathbf{x}_k', t_k)$ is the density function for the transfer of a particle from the point $\mathbf{x}_k'$ to the point $\mathbf{x}$ during the time $t - t_k$ as a result of its inertial motion and the displacement with the mean relative velocity of the phases.

It is evident from Eqs. (7) and (8) that the statistical properties of the velocity fluctuations of the carrier phase on the particle paths are given by the fluctuation characteristics of the fluid particles whose trajectories intersect the trajectory of an isolated particle. We make use of the method of independent averaging to find the two-point correlation of the fluid-phase velocity fluctuations along the particle trajectories in Eqs. (7). This method is widely used in the study of the turbulent diffusion of passive and inertial particles [8-10, 18-21]. As a result, we obtain the following expressions for fluctuation intensity and the eddy diffusion coefficient of the disperse phase:

$$\sigma_{ij}(\mathbf{x}) = \frac{D_0}{\tau} + \frac{1}{\tau^2}\int d\mathbf{x}_1'\int d\mathbf{x}_2'\int_0^t dt_1 \exp\left(-\frac{t-t_1}{\tau}\right)\times$$
$$\times\int_0^t dt_2 \exp\left(-\frac{t-t_2}{\tau}\right)\langle\langle u_i(\mathbf{x}_1', t_1)\, u_j(\mathbf{x}_2', t_2)\rangle\rangle\,\langle G_2(\mathbf{x}, t|\,\mathbf{x}_1', t_1; \mathbf{x}_2', t_2)\rangle, \tag{9}$$

$$D_{ij}^p(\mathbf{x}) = \tau\sigma_{ij} + \int d\mathbf{x}_1'\int_0^t dt_1\left[1-\exp\left(-\frac{t-t_1}{\tau}\right)\right]\times$$
$$\times\langle\langle u_i(\mathbf{x}, t)\, u_j(\mathbf{x}_1', t_1)\rangle\rangle\,\langle G_1(\mathbf{x}, t|\,\mathbf{x}_1', t_1)\rangle,$$

$$\langle G_2(\mathbf{x}, t|\,\mathbf{x}_1', t_1; \mathbf{x}_2', t_2)\rangle = \langle G_1(\mathbf{x}, t|\,\mathbf{x}_1', t_1)\, G_1(\mathbf{x}, t|\,\mathbf{x}_2', t_2)\rangle,$$

where the two-point correlation of fluid velocity fluctuations along the particle trajectory has the form

$$\langle\langle u_i(\mathbf{x}_1', t_1)\, u_j(\mathbf{x}_2', t_2)\rangle\rangle = \langle u_i(\mathbf{x}_1' - \mathbf{R}_f^p(t-t_1), t_1)\, u_j(\mathbf{x}_2' - \mathbf{R}_f^p(t-t_2), t_2)\rangle. \tag{10}$$

For low-inertia particles whose dynamic relaxation time is shorter than the integral macroscopic time scale of the turbulence T_L (T_L is the Lagrangian of the macroscale of the turbulent pulsations), the trajectory of a discrete particle is close to the trajectory of a fluid mole, and Eqs. (8) and (10) describe the Lagrangian correlation of the fluid particles. For inertial particles, $\tau \gg T_L$. Due to the weak correlation between the displacements of the solid and fluid particles, Eqs. (8) and (10) represent the Eulerian two-point correlation of the velocity fluctuations of the carrier phase. Meanwhile, the scales of this correlation function are determined in a coordinate system which moves with the average velocity of the carrier flow. Thus, with an increase in the inertia of the particles, there is a transition from Lagrangian to Eulerian carrier-phase characteristics on the particle trajectory. This conclusion is consistent with results obtained earlier [22].

It should be noted that a local-equilibrium approximation to calculate the fluctuation characteristics of the particles is obtained from Eqs. (9) with $\langle G_1\rangle \sim \delta(\mathbf{x} - \mathbf{x}_1')$ and $\langle G_2\rangle \sim \delta(\mathbf{x} - \mathbf{x}_1')\,\delta(\mathbf{x} - \mathbf{x}_2')$:

$$\sigma_{ij}(\mathbf{x}) = \frac{D_0}{\tau} + \frac{1}{\tau^2}\int_0^t dt_1 \exp\left(-\frac{t-t_1}{\tau}\right)\int_0^t dt_2 \exp\left(-\frac{t-t_2}{\tau}\right)\times$$
$$\times\langle\langle u_i(\mathbf{x}, t_1)\, u_j(\mathbf{x}, t_2)\rangle\rangle,$$

$$D_{ij}^p(\mathbf{x}) = D_0 + \int_0^t dt_1 \langle\langle u_i(\mathbf{x}, t)\, u_j(\mathbf{x}, t_1)\rangle\rangle.$$

With statistically steady turbulence, the two-point correlation function of the carrier-phase velocity fluctuations has the form

$$\langle u_i(\mathbf{x}_1, t_1)\, u_j(\mathbf{x}_2, t_2)\rangle = R_{ij}(\mathbf{x}_1, \mathbf{x}_2; |\mathbf{x}_1 - \mathbf{x}_2|, |t_1 - t_2|)\,\langle u_i u_j\rangle.$$

The transfer density function in the space $G_1(\mathbf{x}, t|\,\mathbf{x}_k', t_k)$ corresponds to the density function for the transfer of a particle having the velocity v_k at point $\mathbf{x}_k'$ at the moment of time t_k to the point $\mathbf{x}$ by the moment t: $G_1'(\mathbf{x}, t|\,\mathbf{x}_k', v_k, t_k)$. The expression for G_1' is obtained from (6) by integrating the function G_0' over $\mathbf{v}$ in the phase space:

$$G_1'(\mathbf{x},\ t|\mathbf{x}_k',\ \mathbf{v}_k,\ t_k) = \delta\Bigg(\mathbf{x} - \mathbf{x}_k' - \tau\mathbf{v}_k\left(1 - \exp\left(-\frac{t - t_k}{\tau}\right)\right) + \int_{t_k}^{t} ds \exp\left(-\frac{t - s}{\tau}\right)\mathbf{u}(\mathbf{R}_p(s),\ s) - \mathbf{W}(t - t_k)\Bigg),\quad k = 1,\ 2. \tag{11}$$

The transfer density functions in the space $\langle G_1\rangle$ and $\langle G_2\rangle$ in Eqs. (9) are determined from the following relations:

$$\begin{aligned} \langle G_1(\mathbf{x},\ t|\mathbf{x}_k',\ t_k)\rangle &= \int d\mathbf{v}_k \langle G_1'(\mathbf{x},\ t|\mathbf{x}_k',\ \mathbf{v}_k,\ t_k)\rangle \langle \Phi(\mathbf{v}_k,\ t_k)\rangle, \\ \langle G_2(\mathbf{x},\ t|\mathbf{x}_1',\ t_1;\ \mathbf{x}_2',\ t_2)\rangle &= \int d\mathbf{v}_1 \int d\mathbf{v}_2 \langle G_1'(\mathbf{x},\ t|\mathbf{x}_1',\ \mathbf{v}_1,\ t_1) \times \\ &\times G_1'(\mathbf{x},\ t|\mathbf{x}_2',\ \mathbf{v}_2,\ t_2)\rangle \langle \Phi(\mathbf{v}_1,\ t_1;\ \mathbf{v}_2,\ t_2)\rangle, \end{aligned} \tag{12}$$

where $\langle\Phi_1\rangle$ and $\langle\Phi_2\rangle$ are the density functions for the velocities of a single particle and a pair of particles at the moments of time t_1 and t_2:

$$\begin{aligned} \langle \Phi_1(\mathbf{v}_k,\ t_k)\rangle &= \langle \delta(\mathbf{v}_k - \mathbf{v}_p(t_k))\rangle,\ k = 1,\ 2, \\ \langle \Phi_2(\mathbf{v}_1,\ t_1;\ \mathbf{v}_2,\ t_2)\rangle &= \langle \delta(\mathbf{v}_1 - \mathbf{v}_p(t_1))\,\delta(\mathbf{v}_2 - \mathbf{v}_p(t_2))\rangle. \end{aligned} \tag{13}$$

To calculate the density functions for the particle velocities, we need to determine the Lagrangian autocorrelation function of the particle-velocity fluctuations

$$\begin{aligned} \langle v_{pi}(t_1)\,v_{pj}(t_2)\rangle &= \frac{1}{\tau^2}\int_0^{t_1} ds_1 \exp\left(-\frac{t_1 - s_1}{\tau}\right)\int_0^{t_2} ds_2 \exp\left(-\frac{t_2 - s_2}{\tau}\right) \times \\ &\times \langle u_i(\mathbf{R}_p(s_1),\ s_1)\,u_j(\mathbf{R}_p(s_2),\ s_2)\rangle = \frac{1}{2}\Bigg[\exp\left(-\frac{t_1 - t_2}{\tau}\right) \times \\ &\times \langle v_{pi}(t_2)\,v_{pj}(t_2)\rangle + \exp\left(\frac{t_1 - t_2}{\tau}\right)\langle v_{pi}(t_1)\,v_{pj}(t_1)\rangle - \\ &- \exp\left(\frac{t_1 - t_2}{\tau}\right)\langle v_{pi}(t_1 - t_2)\,v_{pj}(t_1 - t_2)\rangle\Bigg], \\ \langle v_{pi}(t)\,v_{pj}(t)\rangle &= \frac{1}{\tau}\int_0^{t} ds\left[\exp\left(-\frac{s}{\tau}\right) - \exp\left(-\frac{2t - s}{\tau}\right)\right] \times \\ &\times \langle u_i(\mathbf{R}_p(t),\ t)\,u_j(\mathbf{R}_p(t - s),\ t - s)\rangle. \end{aligned} \tag{14}$$

We will use the method of averaging expressions containing a δ-function to obtain a closed representation of the function $\langle\Phi_2\rangle$. This method was described in [23]. We model particle fluctuation velocity in (13)-(14) as a random normal process. Since the effects of nonlocality are manifest for inertial particles $\tau > T_L$, in examining the particles' relative motion we will limit ourselves to terms of the order $(T_L/\tau)^2$. As a result, we find:

$$\begin{aligned} \langle \Phi_2(v_1,\ t_1;\ \mathbf{v}_2,\ t_2)\rangle &= (2\pi A^2)^{-3/2}\exp\left[-\frac{(v_{1i} - v_{2i})^2}{2A_i^2}\right] \times \\ &\times (2\pi B^2)^{-3/2}\exp\left[-\frac{(v_{1i} + v_{2i})^2}{8\langle v_{pi}^2(t_c)\rangle}\right], \\ A^2 = \prod_{i=1}^{3} A_i^2,\ A_i^2 &= \langle v_{pi}^2(t_s)\rangle\left[1 - \exp\left(-\frac{\xi}{\tau}\right)\right]^2 + \langle v_{pi}^2(\xi)\rangle, \\ B^2 = \prod_{i=1}^{3}\langle v_{pi}^2(t_c)\rangle,\ t_c &= (t_1 + t_2)/2,\ \xi = t_1 - t_2. \end{aligned} \tag{15}$$

The density function for the velocities of a single particle $\langle\Phi_1\rangle$ is obtained from (15) with $\xi = 0(t_1 = t_2)$, $v_1 = v_2$, and has the form of a Maxwell distribution. It is evident from (15) that the mean-square relative velocity of the particles [the parameter A_i in (15)] is composed of the relative velocity of inertial motion and the fluctuation velocity acquired by a particle during the time ξ.

Following the method in [23] and restricting ourselves to an inertial-particle approximation, we can use (11)-(15) to obtain an expression, accurate to within terms of the order $(T_L/\tau)^2$, for the density function $\langle G_2\rangle$ giving the probability of the transfer of two particles from points $\mathbf{x}_1'$ and $\mathbf{x}_2'$ to point $\mathbf{x}$ during the times $t - t_1$ and $t - t_2$, respectively:

$$\langle G_2(\mathbf{x}, t|\mathbf{x}_1', t_1; \mathbf{x}_2', t_2)\rangle = G_2(\mathbf{x}-\mathbf{X}, t-t_c; \mathbf{Y}, \xi) = \\ = (2\pi C^2)^{-3/2}\exp\left[-\frac{(x_i - X_i - W_i(t-t_c))^2}{2L_{pi}^2}\right]\times \\ \times(2\pi\tau^2A^2)^{-3/2}\exp\left[-\frac{(Y_1 - W_1\xi)^2}{2l_{pi}^2}\right], \\ C^2 = \prod_{i=1}^{3} L_{pi}^2,\ L_{pi}^2 = \tau^2\left\{\langle v_{pi}^2(t_c)\rangle\left[1-\exp\left(-\frac{t-t_c}{\tau}\right)\right]^2 + \right. \\ \left. + \langle v_{pi}^2(t-t_c)\rangle\right\},\ l_{pi}^2 = \tau^2A_i^2,\ \mathbf{X} = \frac{\mathbf{x}_1'+\mathbf{x}_2'}{2},\ \mathbf{Y} = \mathbf{x}_1' - \mathbf{x}_2'. \tag{16}$$

The density function for the probability of the transfer of a single particle from point $\mathbf{x}_1'$ to the point $\mathbf{x}$ during the time $t - t_1$ follows from (16) with $\mathbf{x}_1' = \mathbf{x}_2'$, $t_1 = t_2$:

$$\langle G_1(\mathbf{x}, t|\mathbf{x}_1', t_1)\rangle = G_1(\mathbf{Y}_1, \xi_1) = (2\pi C^2)^{-3/2}\times \\ \times\exp\left[-\frac{(Y_{1i} - W_i\xi_1)^2}{2L_{pi}^2}\right],\ \mathbf{Y} = \mathbf{x} - \mathbf{x}_1',\ \xi_1 = t - t_1. \tag{17}$$

It is evident from Eqs. (16)-(17) that the transfer density functions depend on the Lagrangian correlations of the particle-velocity fluctuations. The rate of fluctuation of a particle $\langle v_{pi}^2(t_c)\rangle$ at the moment of time t_c corresponds to the rate of fluctuation of the disperse phase at the point $\mathbf{X}$

$$\langle v_{pi}^2(t_c)\rangle = \sigma_{ii}(\mathbf{X}).$$

For inertial particles $\tau > t_L$, the characteristic spatial scale of change in pulsative motion exceeds the scale of the turbulent moles at point $\mathbf{x}$. In this case, the squares of the fluctuation velocities of the particles $\langle v_{pi}{}^2(\xi)\rangle$ and $\langle v_{pi}{}^2(t - t_c)\rangle$ are determined from Eq. (14). In the latter, the correlation function for carrier-phase velocity fluctuations is calculated at point $\mathbf{X}$ in the locally equilibrium approximation:

$$\langle v_{pi}^2(t')\rangle = \frac{1}{\tau}\int_0^t ds\left[\exp\left(-\frac{s}{\tau}\right) - \exp\left(-\frac{2t'-s}{\tau}\right)\right]\times \\ \times R_{ii}(\mathbf{X}, \mathbf{X}; 0, s)\langle u_i^2\rangle,\ t' = \xi,\ t - t_c. \tag{18}$$

The transfer density function $\langle G_2\rangle$ (16) is the product of two factors. The first is the probability of a transfer occurring along the mean trajectory, beginning at the point $\mathbf{X} = (\mathbf{x}_1' + \mathbf{x}_2')/2$ at the moment of time $t_c = (t_1 + t_2)/2$ and arriving at the point $\mathbf{x}$ at the moment of time t. The second factor is the density function for the distance between two points $\mathbf{x}_1'$ and $\mathbf{x}_2'$ from which the particles arrive at point $\mathbf{x}$. The square of the relative distance between the points $\mathbf{x}_1'$ and $\mathbf{x}_2'$ is calculated from the formula ($x_{1i}' - x_{2i}' = Y_i$):

$$\langle Y_{pi}^2(\xi)\rangle = \int_{-\infty}^{\infty} d\mathbf{X}\int_{-\infty}^{\infty} d\mathbf{Y}Y_i^2 G_2(\mathbf{x}-\mathbf{X}, t_c; \mathbf{Y}, (\xi) = W_i^2\xi^2 + l_{pi}^2. \tag{19}$$

For inertial particles $\tau > T_L$, we find the following from (18) and (19) at $\xi \approx T_E$ (T_E is the time-dependent Eulerian macroscale of turbulence measured in a coordinate system moving with the mean velocity of the flow):

$$\langle Y_{pi}^2(\xi)\rangle = W_i^2\xi^2 + 2\int_0^{\xi} ds(\xi - s)R_{ii}(\mathbf{X}, \mathbf{X}; 0, s) \approx W_i^2T_E^2 + L_E^2,\ \xi\approx T_E, \tag{20}$$

where L_E is the Eulerian scale of the energy moles. Thus, the square of the maximum distance between points x_1' and x_2' from which particles arrive at point x is determined by the phase-slip velocity and the characteristic dimension of the energy moles of the fluid phase.

Taking into account the statistically steady character of the velocity fluctuations of the carrier phase, we rewrite Eq. (9) for the square of the velocity fluctuations of the discrete phase in the form

$$\sigma_{ij}(\mathbf{x}) = \frac{D_0}{\tau} + \frac{\langle u_i u_j \rangle}{\tau^2} \int d\mathbf{X} \int d\mathbf{Y} \int_0^t dt_c \exp\left[-2\frac{t-t_c}{\tau}\right] \times$$
$$\times \int_{-2(t-t_c)}^{2(t-t_c)} d\xi R_{ij}\left(\mathbf{X}+\frac{\mathbf{Y}}{2},\ \mathbf{X}-\frac{\mathbf{Y}}{2};\ |\mathbf{Y}|,\ |\xi|\right) G_2(\mathbf{x}-\mathbf{X},\ t-t_c;\ \mathbf{Y},\ \xi). \tag{21}$$

It is evident from (21) that the maximum value of the integrand in (21) in the variable t_c lies in the region $t - t_c \sim \tau$. It is also apparent that at $t_c = t$, the integral in (21) is equal to zero. Choosing the value of the function G_2 at the point $t - t_c = \tau$ for evaluation of the integral in the variable t_c, we obtain the following ($t \gg \tau$):

$$\sigma_{ij}(\mathbf{x}) = \frac{D_0}{\tau} + \frac{\langle u_i u_j \rangle}{\tau} \int_0^\infty d\xi \exp\left(-\frac{\xi}{\tau}\right) \int d\mathbf{X} \int d\mathbf{Y} \times$$
$$\times R_{ij}\left(\mathbf{X}+\frac{\mathbf{Y}}{2},\ \mathbf{X}-\frac{\mathbf{Y}}{2};\ |\mathbf{Y}|,\ \xi\right) G_2(\mathbf{x}-\mathbf{X},\ \tau;\ \mathbf{Y},\ \xi). \tag{22}$$

The eddy diffusion coefficient for the particles is calculated from the formula

$$D_{ij}^p(\mathbf{x}) = \tau\sigma_{ij} + \langle u_i u_j \rangle \int_0^\infty d\xi \left[1-\exp\left(-\frac{\xi}{\tau}\right)\right] \times$$
$$\times R_{ij}(\mathbf{x},\ \mathbf{x}-\mathbf{Y};\ |\mathbf{Y}|,\ \xi)\, G_1(\mathbf{Y},\ \xi). \tag{23}$$

It follows from Eqs. (16) and (22)-(23) that the fluctuation velocities of the disperse phase are calculated by a self-consistent method. This is in keeping with the results obtained in [8, 9]. It is also evident from (22) and (23) that the fluctuation characteristics of the disperse phase depend on the degree of involvement of the particles in the pulsative motion associated with the small-scale turbulence that forms the internal structure of the turbulent moles. The fluctuation characteristics are also seen to depend on the character of the change in the energy of the turbulent pulsations of the carrier phase in space.

2. To illustrate these relations, we will examine the unidimensional case in a uniform turbulence approximation. The expressions for the fluctuation velocity and the eddy diffusion coefficient (22), (23) take the form

$$\sigma = \frac{D_0}{\tau} + \frac{\langle u^2 \rangle}{\tau} \int_0^\infty d\xi \exp\left(-\frac{\xi}{\tau}\right) \int_{-\infty}^\infty dY R_0(|Y|,\ \xi) \times$$
$$\times (2\pi l_p^2)^{-1/2} \exp\left[-\frac{(Y-W\xi)^2}{2l_p^2}\right], \tag{24}$$

$$D_p = D_0 + \langle u^2 \rangle \int_0^\infty d\xi \int_{-\infty}^\infty dY R_0(|Y|,\ \xi) (2\pi l_p^2)^{-1/2} \exp\left[-\frac{(Y-W\xi)^2}{2l_p^2}\right], \tag{25}$$

$$l_p^2 = \tau^2\left\{\sigma\left[1-\exp\left(-\frac{\xi}{\tau}\right)\right]^2 + \frac{\langle u^2 \rangle}{\tau} \int_0^\xi ds \left[\exp\left(-\frac{s}{\tau}\right) - \right.\right.$$
$$\left.\left. - \exp\left(-\frac{2\xi-s}{\tau}\right)\right] R_0(0,\ s)\right\}. \tag{26}$$

We assign the two-point autocorrelation function for the fluctuations of gas velocity on the particle trajectory in the form

$$R_0(Y,\ \xi) = R_1(Y/L_p')\, R_2(\xi/T_p'),$$

where L_p' and T_p' are the spatial and temporal macroscales of the fluid pulsations on the particle trajectory. The macroscales of the pulsations of the gas on the particle trajectory are connected with the Lagrangian and Eulerian macroscales of the pulsations of the turbulent field of the carrier phase:

$$T_p' = (\omega + 1) T_L/2 + (1 - \omega) T_E/2,$$
$$L_p' = (\omega + 1) L_L/2 + (1 - \omega) L_E/2, \tag{27}$$
$$\omega = (1 - \alpha)/(1 + \alpha),\ \alpha = \langle Y_p^2 (T_p) \rangle^{1/2}/L_L.$$

For inertial particles $\tau > T_L$ in the case when there is an appreciable mean phase-slip velocity $WT_L \gg L_L$, $\alpha \gg 1$ and $\omega \to -1$. In this case, we find from (27) that the scales of the fluctuation field of the carrier phase on the particle trajectory coincide with the Eulerian macroscales measured in a coordinate system moving with the mean velocity of the flow. For low-inertia particles $\tau \ll T_L$ with a negligible mean phase-slip velocity $WT_L \ll L_L$, $\alpha \to 0$, $\omega \to 1$, and the turbulent field around a particle has Lagrangian characteristics.

We will calculate the eddy diffusion coefficient of the particles with a stepped approximation of the autocorrelation function for the gas-velocity fluctuations near a particle:

$$R_0(Y, \xi) = \theta(L_p' - |Y|)\,\theta(T_p' - |\xi|), \tag{28}$$
$$l_p = l_p(T_p),\ \beta = T_L/T_E = L_L/L_E,\ \beta' = T_p'/T_E = L_p'/L_E,$$
$$\Omega = \tau/T_E,\ f = 1 - \exp(-\beta'/\Omega),\ w = W/\langle u^2 \rangle^{1/2},$$
$$\alpha_1 = 1 + w,\ \alpha_2 = 1 - w,\ a = L_p'/(\sqrt{2}\, l_p),$$
$$l_p/L_E = \Omega f (1 + \sigma/\langle u^2 \rangle)^{1/2},$$

$$\frac{D_p - D_0}{D_f} = \frac{\langle u^2 \rangle^{1/2}}{2\beta W}\Big\{\beta' [\alpha_1 \operatorname{erf}(a\alpha_1) - \alpha_2 \operatorname{erf}(a\alpha_2)] + {} + \frac{l_p}{2\sqrt{\pi} L_E}\left[\exp(-a^2\alpha_1^2) - \exp(-a^2\alpha_2^2)\right]\Big\}, \tag{29}$$

where the parameter β describes the relation between the Lagrangian and Eulerian macroscales of the fluctuation field of the carrier phase.

It should be noted that the well-known formulas which consider the effect of intersection of the trajectories on the eddy diffusion coefficient of the particles follow from (25) with the corresponding approximations of the function $R_0(Y, \xi)$.

When $\ell_p \to 0$ and $R_0(Y, \xi) = \exp(-Y^2/L_E^2 - \xi^2/T_L^2)$, we obtain the Csanady formula [13]

$$(D_p - D_0)/D_f = (1 + W^2\beta^2/\langle u^2 \rangle)^{-1/2}. \tag{30}$$

With an exponential approximation $R_0(Y, \xi) = \exp(-|Y|/L_E - |\xi|/T_L)$, we find [4, 12]

$$(D_p - D_0)/D_f = (1 + \beta W/\langle u^2 \rangle^{1/2})^{-1}. \tag{31}$$

Figure 1 compares the results of calculation of the eddy diffusion coefficient of the particles from Eqs. (29)-(31) in relation to phase slip velocity. The calculations were performed with $\beta = 0.8$ [13]. It is evident that an increase in particle inertia (curves 4 and 5) causes a reduction in the eddy diffusion coefficient of the coarse particles. This can be attributed to the lesser involvement of an inertial impurity in the small-scale, high-frequency pulsative motion of the small turbulent eddies which go into the structure of a turbulent mole. With an increase in the relative velocity of the phases, the value of the ratio $W/\langle u^2 \rangle^{1/2}$ becomes the main factor which determines the level of turbulent diffusion of the dispersed impurity. It should be noted that with a decrease in the Lagrangian scales of the turbulent field compared to the Eulerian scales ($\beta \ll 1$), the particles' eddy diffusion coefficient exceeds the eddy diffusion coefficient of the fluid phase. A similar result was observed in [9, 10, 22].

Figure 2 illustrates the change in the fluctuation energy of the particles in relation to inertial parameters and phase slip velocity. The calculations were performed with Eq. (24) for autocorrelation function (29). It follows from the figure that a reduction in the

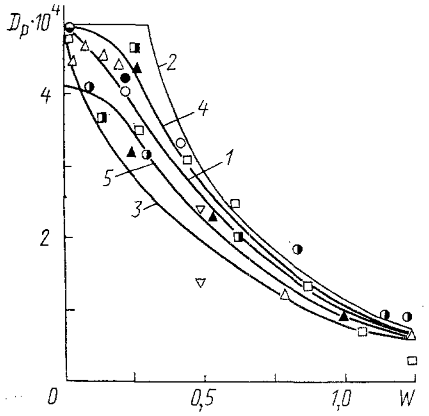

Fig. 1

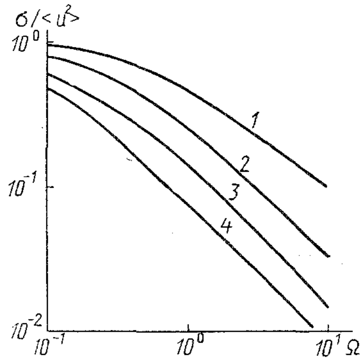

Fig. 2

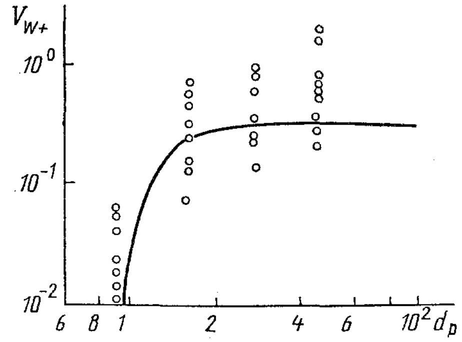

Fig. 3

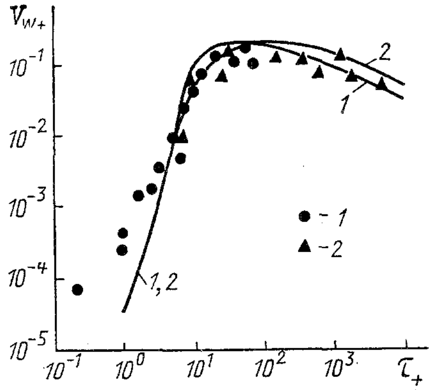

Fig. 4

Fig. 1. Effect of mean phase-slip velocity on the eddy diffusion coefficient of the particles: points) experimental data [14]; 1) calculation with Eq. (30); 2) with Eq. (29) at $\ell_p = 0$; 3) with Eq. (31); 4) with Eq. (29) at $d_p = 15$ μm; 5) with Eq. (29), $d_p = 100$ μm. D_p, m²/sec; W, m/sec.

Fig. 2. Effect of phase slip velocity and particle inertia on particle fluctuation velocity: 1) $W/\langle u^2\rangle^{1/2} = 0$; 2) 5; 3) 10; 4) 20.

Fig. 3. Rate of particle deposition on a channel wall ($u_+ =$ 0.34 m/sec) in relation to particle size (points show experimental data from [25]). d_p, μm.

Fig. 4. Effect of particle inertia on deposition rate (points show experimental data from [26]): 1) Re = 10^4; 2) $5\cdot10^4$.

fluctuation energy of the impurity is greater, the greater the inertia of the particles and the relative phase-slip velocity.

3. Let us evaluate the validity of the expressions obtained here to determine the fluctuation velocity of the discrete phase (22) in the case of nonuniform turbulence occurring in the wall region of a channel on a stabilized section of the flow. The literature contains little empirical data on particle fluctuation velocity near channel walls, while there are data on the rate of particle deposition on walls during the turbulent flow of a gas suspension [25-27]. It follows from the results in [17] that for an absolutely absorbing channel wall, the rate of particle deposition is connected with the intensity of the transverse pulsations of the disperse phase by the following relation:

$$(2/\pi)^{1/2}\sigma^{1/2}(0) = V_W, \tag{32}$$

where σ(0) is the square of the transverse pulsations of particle velocity on the wall; V_W is the rate of particle deposition.

We approximate the autocorrelaton function for pulsations of gas velocity in the transverse direction by means of the expression

$$R(y_1, y_2; y_1-y_2, \xi) = \langle u^2\left(\frac{y_1+y_2}{2}\right)\rangle R_0(y_1-y_2, \xi),$$
$$\langle u^2(y)\rangle = bu_+^2[1-\exp(-y_+/A_+)]^2, \quad y_+ = yu_+/\nu, \tag{33}$$

where $A_+ \approx 30$; $b \sim 1$ [24]; u_+ is dynamic velocity; R^0 is assigned in the form (28). We evaluate the spatial and temporal macroscales of the turbulent pulsations of the gas for the core of the flow: $L_E \approx \gamma R$, $T_E \approx L_E/u_+$, $\gamma = 0, 1$.

Inserting (2) and (33) into (22) and integrating, we find an expression for the fluctuation velocity of the disperse phase on the channel wall. At W = 0, this expression has the form

$$\frac{\sigma(0)}{u_+^2} = \frac{b}{2} f_1\left[1 - 2\exp\left(\frac{L_{p+}^2}{2A_+^2}\right)\operatorname{erfc}\left(\frac{L_{p+}}{\sqrt{2}A_+}\right) + \right.$$
$$\left. + \exp\left(\frac{2L_{p+}^2}{A_+^2}\right)\operatorname{erfc}\left(\frac{\sqrt{2}L_{p+}}{A_+}\right)\right], \quad L_{p+} = \frac{L_p u_+}{\nu}, \tag{34}$$
$$f_1 = f\operatorname{erf}\left(\frac{L_p'}{\sqrt{2}l_p}\right), \quad L_p^2 = a_1\tau^2(\sigma + f_1\langle u^2\rangle),$$
$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x), \quad a_1 \approx 0.5.$$

With $\beta = 0.3$ and $b = 0.8$, we used Eqs. (32) and (33) to calculate rates of particle deposition on the surface. Figure 3 compares the results of calculation of particle deposition rate in the flow of a dust-laden gas past a plate. Figure 4 illustrates the effect of the dimensionless relaxation time of the particles on deposition rate. An increase in the inertia of the particles ($\tau_+ < 10^2$) leads to intensive penetration of the dispersed impurity into the viscous sublayer and an increase in the fluctuation energy of the impurity on the walls. Coarser particles ($\tau_+ > 10^2$) are drawn into pulsative motion in the flow core to a lesser extent, which results in a reduction in the turbulence energy of the impurity over the entire cross section of the channel.

NOTATION

τ, dynamic relaxation time of particles; σ_{ij}, second moments of particle-velocity fluctuations; $\langle U_i(x, t)\rangle$, $u_i(x, t)$, mean and fluctuation components of carrier-flow velocity; W_i, mean velocity of phase slip caused by body forces; $f_i(x, t)$, random field describing Brownian fluctuations of particle velocity; $V_{pi}(t)$, $v_{pi}(t)$, actual and fluctuation velocities of a single solid particle; D_0, coefficient of Brownian diffusion of particles; D_p, coefficient of eddy diffusion of particles; $\delta(x)$, Dirac delta function; $\langle V_i(x, t)\rangle$, averaged component of particle velocity; $R_f{}^P$, coordinate of a fluid particle on the trajectory of a discrete particle; $\langle G_1\rangle$, $\langle G_2\rangle$, transfer density function for one and two particles in space; D_f, coefficient of eddy diffusion of the carrier phase; $\theta(x)$, Heaviside function; $\operatorname{erf}(x) = 2/\sqrt{\pi} \times \int_0^\infty dt\exp(-t^2)$, standard error function; L_E, T_E, Eulerian temporal and spatial macroscales; d_p, particle diameter; $Re = 2RW_m/\nu$, Reynolds number of flow; R, channel radius; u_+, dynamic velocity; $\tau_+ = \tau u_+{}^2/\nu$, particle relaxation time in dynamic variables; $y_+ = yu_+/\nu$, distance from channel wall in dynamic variables; $V_{W+} = V_W/u_+$.

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HEAT RELEASE IN A GAS FLOW AT THE JUNCTION OF CHANNELS WITH DIFFERENT SURFACE ROUGHNESSES

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UDC 533.6.011

A measurement is made of the thermal polarization which develops in a gas flow in a nonuniform channel due to the dependence of the mechano-caloric heat flux on surface roughness.

In accordance with nonequilibrium thermodynamics, in a compound channel or V-shaped pipe with parts having different surface roughnesses, a heat flux should develop on the wall J_q in the region of the junction of dissimilar parts when an isothermal gas flows through the pipe [1]. This effect is related to the accommodation pumping effect observed by Hobson [2]. The latter phenomenon involves the formation of a longitudinal flow of gas particles when the temperatures of the walls at the junction of dissimilar channels deviate from the temperatures of their free ends. Measurement of the heat flux J_q is of scientific interest, since it makes it possible to prove hypotheses regarding the nonequilibrium thermodynamics of discontinuous systems and determine the value of the kinetic coefficient L_{qp}, which characterizes the release of heat in the region of the joint of dissimilar parts of a compound channel.

The authors of [3] proposed a theoretical model to calculate the above-mentioned effect for small Knudsen numbers. The method is based on the solution of problems of continuum mechanics with the use of slip boundary conditions.

We write as follows the temperature and pressure fields in the flow of a gas in a long ($L \gg R$) isothermal nonuniform channel when the conditions correspond to the viscous slip

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